A SAILPLANE WING CONSTRUCTED OF FOAM CORE AND POLYESTER FIBREGLASS SKIN

By

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1973

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SUMMARY

The results from a stress analysis of a thin skin, foam core, high aspect ratio wing indicate a possible method of constructing sailplane wings. The analysis includes an approximation of the maximum core and skin shear stress, a computer program to evaluate the stress distribution and displacements of a thin walled unsymmetrical tapered cylinder, and the accountability of creep.
Composite beams are designed so that each material in the beam is most efficiently used with respect to weight, position, and resistance to forces.

Take for example the beam shown in Figure 2. The flange material is positioned so that it resists the maximum normal and shear stresses. Therefore a high modulus material should be used. The core material is positioned so that it spaces the flange material and resists the maximum shear stress. A lighter weight material is chosen for the core material because it occupies the most volume of the beam. But the lighter weight core materials usually have lower allowable maximum normal and shear stresses. This is of no consequence because the core’s normal stress is small ($E_{\text{core}} \ll E_{\text{skin}}$) and the core’s maximum allowable shear stress is within working limits. So the only two limitations are the flange’s maximum allowable normal stress and the core’s maximum allowable shear stress. With these two design requirements a beam can be designed with a tolerable reduction in maximum strength and a substantial savings in weight.

The construction of most high aspect ratio sailplane wings today include one spar, ribs, and a thin skin, Figure 3A. Most of the high performance European sailplanes have wings constructed of spar and thick skin with no ribs, Figure 3B. The skin has a composite sandwich construction with a foam core laminated between layers of resin and glass cloth. The spar is usually constructed from epoxy resins and glass fabric. In such a construction the
skin resists most of the normal forces and the shear webs in the spar resists some of the shear forces. The shear webs are kept thin as in an I-beam to make efficient use of weight. The idea of using a different type of shear web such as foam core is possible so long as the same shear stress requirements is satisfied. Buckling of the skin is at present resisted by the thick laminated skin. The thick box beam spar also resists buckling of the thick skin because of the short chord. The use of a foam core would also resist buckling. Therefore the use of a low density foam core instead of a thin higher density shear web is completely with in reason. And such is the objective of this senior project, to structurally evaluate a high aspect ratio wing constructed of foam core and resin-glass skin.
\[ \begin{align*}
E_c & \quad \text{Modulus Of Elasticity For The Core} \\
 b & \quad \text{Width Of The Composite Beam Crosssection} \\
 d & \quad \text{Total Thickness Of Composite Beam} \\
 \mu & \quad \text{Poisson's Ratio} \\
 \gamma_{\text{max}} & \quad \text{Maximum Shear Stress} \\
 \gamma_{\text{cmax}} & \quad \text{Maximum Core Shear Stress} \\
 \gamma_{\text{smax}} & \quad \text{Maximum Skin Shear Stress} \\
 t_{\text{shear}} & \quad \text{Skin Thickness That Resists} \quad \gamma_{\text{smax}} \\
 \sigma_{\text{ult}} & \quad \text{Skin Ultimate Normal Stress} \\
 t_{\text{normal}} & \quad \text{Skin Thickness That Resists} \quad \sigma_{\text{ult}} \\
 \sigma_w & \quad \text{Wrinkling Stress} \\
 B_1 & \quad \text{Constant} \\
 \rho & \quad \text{Constant} \\
 t_s & \quad \text{Skin Thickness For Favorable Wing Deflections} \\
 t_w & \quad \text{Skin Thickness That Resists} \quad \sigma_w \\
 t & \quad \text{Skin Thickness} \\
 E_s & \quad \text{Modulus Of Elasticity For The Skin} \\
 M & \quad \text{Bending Moment} \\
 t_c & \quad \text{Core Thickness} \\
 w_c & \quad \text{Core Width}
\end{align*} \]
The wing crosssection was defined as a NACA 4412. The root and tip chord are 8.5 and 4.25 inches respectively with no sweep back at the leading edge, Figure 1. The span was 51.1875 inches with a 17.7 aspect ratio. This particular wing had the same dimensions as a wooden wing used on one of the authors radio-controlled sailplanes. The small size made it practical to build and hopefully at some future time fly.
SKIN-CORE APPROXIMATIONS

As shown in Appendix A the flexural rigidity (bending modulus) of a composite beam shown in Figure 2 is:

\[
(EI)_{\text{composite}} = \frac{E_s b}{12 (1-\mu^2)} \left[ d^3 - t_c^3 \left(1 - \frac{E_c}{E_s}\right) \right]
\]

(1)

For beams in bending the deflection equation shows that the deflections will depend on the combined rigidity of the skin and core materials.

If \( E_c \ll E_s \) then the flexural rigidity is seen to depend only on the skin modulus of elasticity.

\[
(EI)_{\text{composite}} = \frac{E_s b}{12 (1-\mu^2)} \left[ d^3 - t_c^3 \right]
\]

(3)

The percent of deflection due to shear was assumed to be small and therefore neglected, Appendix C.

Unfortunately the core cannot be totally ignored. Although the core does not resist bending it does resist a part of the maximum shear stress and resists buckling of the thin skin.
MAXIMUM SHEAR STRESS

If the airfoil shape is simplified as a rectangular composite beam the formulas used to calculate shear stresses can be used, Appendix D. The equation for the maximum shear stress of a thin skin foam core was derived,

$$\gamma_{\text{max}} = \gamma_{c,\text{max}} + \gamma_{s,\text{max}} = \frac{V_{\text{max}}}{8bt + 4(2t + W_c \frac{E_c}{E_s})} \left[ \frac{2(\ell + d)bt + t_c^3(2t + W_c \frac{E_c}{E_s})}{b(d^3 - t_c^3) + t_c^3(2t + W_c \frac{E_c}{E_s})} \right] / 2$$

Using the method of equivalent areas a relation for the shear forces in the core and skin was formulated, Appendix D,

$$\gamma_{s,\text{max}} : \gamma_{c,\text{max}} = 5:3 \quad (5)$$

for the airfoil shape at midspan.

By approximating the dimensions of the airfoil at midspan as a rectangular composite beam the equations (4) and (5) were solved simultaneously for core and shear stresses at a maximum shear load of 10 lbs. (7G's).

$$\gamma_{c,\text{max}} = 3.33 \text{ lb/in}^2 \quad \gamma_{s,\text{max}} = 177 \text{ lb/in}^2$$

Skin thickness necessary to resist $\gamma_{\text{max}}$, Appendix D.

$$t_{\text{shear}} = 0.025 \text{ in.}$$
MAXIMUM NORMAL STRESS

An approximate maximum normal stress at a 70 wing loading is calculated by using the elastic flexural formula. The root airfoil

\[ \sigma_{\text{ultimate}} = 6240 \text{ psi} \quad t_{\text{Normal}} = 0.0091 \text{ in.} \]

is approximated as a rectangular cross-section so that the moment of inertia can be found as a function to skin thickness, Appendix E.
SKIN WRINKLING

A method for determining skin thickness required to prevent wrinkling of skin as a function of core and skin properties was outlined by Knight.¹

The compressive stress in the facing material at which wrinkling will occur is given by:

\[ \sigma_w = B_1 \frac{E_s}{E_c}^{1/3} \frac{E_c}{E_s}^{2/3} \]  

(6)

The constant \( B_1 \) is plotted as a function of \( \rho \) where \( \rho \) is given by:

\[ \rho = \frac{t_s}{t_c} (\frac{E_s}{E_c})^{1/3} \]  

(7)

For values of \( \rho \geq 0.25 \), \( B_1 \) is a constant, \( B_1 \approx 0.575 \).

\[ \sigma_w = 0.575 \frac{E_s}{E_c}^{1/3} \frac{E_c}{E_s}^{2/3} \]  

(8)

Substituting the skin and core modulus of elasticity into (8) gives a wrinkling stress.

\[ \sigma_w = 5475 \text{ psi} \]
Considering this as a maximum normal stress. The skin thickness can again be determined as in Appendix B.

\[ t_w = 0.011 \text{ m.} \]
The fact that the core properties could be ignored in the composite beams deflections motivated the stress analysis of a thin walled-unsymmetrical crosssection-tapered cylinder.

A computer program was written to take any defined shaped thin walled cylinder (wing shape) and give deflections and twists along the span for any defined wing loading, Appendix F.

The program was tested for accuracy by defining a cylindrical shape whose deflections, moments of inertia, shear center location, weight, etc. were compared with hand calculations. The program was found to be very accurate for the thin skin approximation.
The wing was approximated as a rectangular box with a uniform distributed load simulating different G loads, Appendix G. Skin thicknesses sufficient for resisting $\sigma_{\text{ultimate}}$ and buckling produced intolerably large deflections. Therefore the skin thickness was increased so that deflections reasonable for aero-elastic effects would be realized. A final skin thickness was calculated to give reasonable deflections for a 7G simulated load and resist the

$$t_s = 0.025 \text{ in}$$

maximum shear stress.
WING FABRICATION

With regard to money and time the wing was not constructed with the best material or fabrication procedures. Instead relatively inexpensive materials and simplified fabrication methods were used to construct a testable wing. The skin thickness was not closely controlled so long as the thickness was known and related deflections measurable.

The foam/core was constructed of CPR 9005-2 rigid urethane foam. Originally it was hoped that the foam core shape could be cut by a hot wire guided over root and tip airfoil templates. Unfortunately it was found difficult to cut urethane foam sections longer then a few feet. The urethane core was therefore shaped by sanding spanwise with the root and tip airfoil templates as guides.

The skin was fabricated by wet lay up of 6 ounce glass cloth and polyester resin. The surface was sanded once and a finish coat applied.

A one inch wide hard wood root airfoil shape was glued to the foam and was also covered by resin and glass. This served as a noncrushable rigid support for the fixed condition of a cantilever beam, Figure 4C, 4B.
TEST PROCEDURE

The wing was mounted in a fixed condition at the root, Figure 4, with the flat side of the airfoil facing upward. The flat side of the wing was leveled so that an evenly distributed load could be applied on the flat surface in a predictable manner. Time and presence of creep permitted only a 1G and 2G load for measuring deflections, Table I.

Wing G loading was simulated by a uniform distributed load using a string of metal slugs attached to a line of tape. The slugs were measured and distributed to simulated a 1G and 2G load. The load was applied to a line drawn from the root aerodynamic center to the tip aerodynamic center.

The wings airfoil shape at the root and tip were measured, Table II. The skin thickness along the span was measured, Table III. A piece of the skin was removed from the wing and tensile tested for a modulus of elasticity, Table V. The above was substituted into the computer program to obtain the calculated deflections which are compared with the experimental deflections in Table I.
The deflections were calculated assuming the core was insignificant in resisting bending. The modulus of the core and skin were experimentally measured and the term

\[
(1 - \frac{E_c}{E_s})
\]

in the composite flexural rigidity equation was found to be so nearly equal to one that the core had little or no effect on the composite beams deflections, Appendix B. The experimental deflections verified that this was a good approximation, Table I. The percent error in deflections are a combination of, the core materials contribution to resisting bending, accuracy of deflection measurements, creep, and numerical methods used to calculate deflections. The core's contribution in resisting deflection, although shown to be small, account for the smallest of the possible error sources. Creep is the largest contributor, since the percent errors are seen to be higher for the larger G loads. The error in measuring the deflections with a 1 inch travel 0.001 inch dial gage was less then 1%. The error in the numerical methods is shown to be less then 1%, Appendix F.

The presence of creep was noted, Table IV. For the wing being tested, little should be said about the creep since the fabrication methods and materials were not the optimum. But creep can and should be accounted for in reinforced plastics who's materials and
fabrication are more closely controlled. A method for determining
time, temperature, and rupture stresses in reinforced plastics shows
that creep in reinforced plastics can withstand large stresses for
long periods of time at room temperatures.\textsuperscript{3} For example from a series
of tests Plaskon 920 (a polyester resin-glass laminate) can resist
a stress of 28,500 psi for more then five years before rupture.\textsuperscript{3} It
was found that these long stress to rupture estimates could be accur-
ately predicted by a relation derived by Larson and Miller.\textsuperscript{3}

\[ T \left( 20 + \log t \right) = \text{constant} \]

So that long-time low temperature creep results could be calculated
from data of short-time high temperature creep tests.

The shear stress distribution used to calculate the shear center
location for a thin walled unsymmetrical tapered cylinder is question-
ably used for the composite wing since it has already been shown that
the core material resists a substantial portion of the total shear load,
Appendix D.

The normal skin stresses calculated by the computer program
accurately predicts the real stress experienced by the composite
wing since it has already been shown that the skin resist almost
all the bending in the beam.

The foam core as was previous shown resist only the maximum
core shear stress. Unfortunately the lowest density foam core has
an ultimate shear stress much greater than the maximum core shear
stress. A lower density foam could be used so that a lighter wing
will result without lowering the ultimate shear stress past the maximum core shear stress. As of now the wing in comparison with an identical wing made of wood is about twice as heavy.

It can be shown that the modulus of elasticity for glass-fiber-reinforced polyester resins vary a small amount with respect to the orientation of the glass cloth weave.
CONCLUSIONS

The computer program used to calculated deflections and normal stresses for the thin skin only, are the same deflections and skin stresses existing for the composite (skin and foam) so long as the term

\[(1 - \frac{E_c}{E_s})\]

in the flexural rigidity of a composite is approximately equal to one.

Creep in glass reinforced plastics at room temperatures can withstand large stresses for long periods of time before rupture.

Skin thicknesses adequate to resist the maximum normal stresses and buckling produced inadequately large deflections. The skin thickness depends only on the desired magnitude of deflections and the maximum skin shear stress.

The foam core resist only the maximum core shear stress and therefore the lowest density necessary to safely resist this shear stress should be used.
RECOMMENDATIONS

It is now possible with the established methodology to optimize such a wing with respect to weight and strength. Time did not permit optimization of the composite wing and the lack of information on the construction of present sailplane wings did not justify a comparison. The author feels that both points should be covered before any large wing fabrication is considered.

At best this senior project demonstrates that such a wing is possible to analyze, build, and could possibly be competitive with the present high aspect ratio sailplane wings.

As already pointed out the shear center location, although accurately calculated for the thin skin only, was not proven to be accurate for the composite case. Specially shaped composite beams can be constructed so that the empirical shear center location for the composite beam could be compared with the location calculated by the skin approximation method.
REFERENCES


FIGURES AND TABLES
FIGURE 1
WING DIMENSIONS

SPAN 51.1875 in.
A.R. 17.7
SKIN THICKNESS 0.030

SCALE 1:1
\[ \gamma_{\max} = \frac{V Q}{I b} = \frac{M_{\max}}{b(t_c + d)} \]
### Table I: Experiment Data

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**1G Lift**

**Uniform Lift Distribution** 0.027 lb/in

**2G Lift**

**Uniform Lift Distribution** 0.054 lb/in

Deflections measured by 0.001 inch increment 1 inch travel dial gage.
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<td>----------------------------------</td>
<td>------</td>
<td>------</td>
</tr>
<tr>
<td>SKIN THICKNESS inches</td>
<td>0.031</td>
<td>0.028</td>
</tr>
<tr>
<td>DENSITY, lb/in$^3$</td>
<td>-</td>
<td>0.0351</td>
</tr>
</tbody>
</table>
# Table IV

## Creep

<table>
<thead>
<tr>
<th>TIME, minutes</th>
<th>0</th>
<th>10</th>
<th>30</th>
</tr>
</thead>
<tbody>
<tr>
<td>TIP DEFLECTIONS (inches)</td>
<td>0.550</td>
<td>0.615</td>
<td>0.640</td>
</tr>
</tbody>
</table>

## Relaxation

24 hour period, tip deflection 0.005 in
<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>MODULUS OF ELASTICITY psi</th>
<th>ULTIMATE NORMAL STRESS psi</th>
<th>ULTIMATE SHEAR STRESS psi</th>
<th>DENSITY lb/ft³</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPR 9005-2 URETHANE FOAM</td>
<td>14415</td>
<td></td>
<td></td>
<td>6.6 x 10⁻²</td>
</tr>
<tr>
<td>POLYESTER GLASS REINFORCED SKIN</td>
<td>6240</td>
<td></td>
<td></td>
<td>0.00116</td>
</tr>
</tbody>
</table>

*SKIN WAS TENSILE TESTED

SPECIMEN - GAGE LENGTH 6 inches
            - WIDTH 1 inch
            - GLASS FABRIC ORIENTATION 30 degrees from longitudinal axis
Ultimate Load: 190 lbs
Ultimate Stress: 6240 psi

Cross Head Feed: 0.02 inches/min
Chart Speed: 2 inches/min

Area, $A = 0.030 \text{ m}^2$

$6'' = \text{Gage Length}$

$F = \frac{P}{A} = 1100 \text{ psi}$

$\epsilon = \frac{\delta}{L} = 0.00167$
APPENDIX A

FLEXURAL RIGIDITY OF A COMPOSITE BEAM
DERIVATION OF THE FLEXURE RIGIDITY OF A COMPOSITE BEAM

$E_c$ Modulus of elasticity of the core in the span wise direction

$E_s$ Modulus of elasticity of the skin

$t_c$ Thickness of core

$d$ Total thickness

$b$ Length of panel edge

$\nu_s$ Poisson's ratio of plate

$I_s$ Moment of inertia of the skin only

$D$ Flexure rigidity of the composite beam

A quick first approximation

$$D \approx E_s I_s = \frac{b E_s}{12} \left[ d^2 - t_c^2 \right]$$

A better approximation can be made if we assume that a beam is very wide compared to the depth then the lateral deformation is prevented and the beam is stiffer. In our case, we have two face plates whose width $\gg$ thickness.
Consider a wide beam in pure bending.

\[ \Delta x = r \Delta \theta \]
\[ \delta = (r + v) \Delta \theta - \Delta x \]
\[ \delta = v \Delta \theta \]
\[ \varepsilon_x = -\frac{\delta}{\Delta x} = -\frac{v}{r} \]

Stress occurs on an element of length \( \Delta x \) located a distance \( v \) from the neutral axis.

Since lateral movement is prevented therefore \( \varepsilon_y = 0 \)

Then \( S_y \) is induced in the material to keep sides straight and parallel.

From Hooke's law assuming \( s_z = 
\varepsilon_{yz} = \gamma_{xz} = 0 \)

\[ S_x = \frac{E}{1 - \mu^2} \left( \varepsilon_x + \mu \varepsilon_y \right) = \frac{E}{(1 - \mu^2)} \gamma \]
\[ M = \int_{\text{area}} S x y \, dA \quad \frac{E_b}{(1-\mu^2)r} \int_{-d/2}^{+d/2} y^2 \, dv = \frac{E_b d^3}{12(1-\mu^2)r} \]

Recalling the basic relations, \( \frac{1}{r} = \frac{M}{EI} \), \( D = EI \)

\[ D = \frac{E_b d^3}{12(1-\mu^2)} \]

For two face plates separated by core material,

\[ D = \frac{E_b b d^3}{12(1-\mu^2)} - \frac{E_b b t_c^3}{12(1-\mu^2)} = \frac{E_b b}{12(1-\mu^2)} \left[ d^3 - t_c^3 \right] \]

The idea of adding or subtracting the property of EI for various sections from the original whole crosssection can give an even better approximation.

\[ D_{\text{core}} = \frac{E_b b t_c^3}{12(1-\mu^2)} \]

\[ D = \frac{E_b b d^3}{12(1-\mu^2)} - \frac{E_b b t_c^3}{12(1-\mu^2)} + \frac{E_b b t_c^3}{12(1-\mu^2)} \left[ d^3 - t_c^3 \left(1 - \frac{E_c}{E_b} \right) \right] \]
APPENDIX B

CORE MODULUS
CORE MODULUS

The core modulus was calculated by measuring the deflections of a slab of CPR 9005-2 rigid urethane foam.

Dimensions: Length in. 56
Width in. 10
Depth in. 1.75

\[ I_c = \text{Moment of inertia of the beam} = 4.47 \text{ m}^4 \]

<table>
<thead>
<tr>
<th>LOAD P at midspan lb.</th>
<th>DEFLECTION ( \delta ) at midspan inch.</th>
<th>( \frac{P}{\delta} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.135</td>
<td>0.078</td>
<td>1.73</td>
</tr>
<tr>
<td>0.270</td>
<td>0.150</td>
<td>1.80</td>
</tr>
<tr>
<td>0.540</td>
<td>0.327</td>
<td>1.66</td>
</tr>
</tbody>
</table>

\[ \frac{P}{\delta} \text{ avg} = 1.73 \]

\[ E_c = \frac{P}{\delta} \frac{L}{48 I_c} = 1419 \]

We can now evaluate the term \( \left( 1 - \frac{E_c}{E_s} \right) \)

From Table I \( E_s = 6.6 \times 10^5 \)

\[ \left( 1 - \frac{E_c}{E_s} \right) = 0.99785 \]
APPENDIX C

DEFLECTIONS DUE TO BENDING AND SHEAR
DEFLECTIONS DUE TO BENDING AND SHEAR

\[ M = \frac{3}{3} \left( \frac{2x\sqrt{H^2x/L}}{3} \right) + Q \times \]

\[ V = \frac{2x\sqrt{H^2x/L}}{3} + Q \]

\[ \frac{\delta M}{\delta Q} = x \]

\[ \frac{\delta V}{\delta Q} = +1 \]

\[ U = \int_{0}^{L} \frac{1}{2} \frac{M^2}{EI} \, dx + \int_{0}^{L} \frac{1}{2} \frac{V}{GA} \, dx \]

\[ S_a = \frac{3U}{3Q} = \int_{0}^{L} \frac{M}{EI} \, dx + \int_{0}^{L} \frac{3M}{2Q} \, dx \]

\[ S_a = \frac{1}{EI} \int_{0}^{L} \left[ \frac{3x}{5} \left( \frac{2x}{3} \right) \left( \frac{H^2x}{L} \right)^2 + Qx \right] \, dx + \frac{1}{GA} \int_{0}^{L} \left[ \frac{2x}{3} \left( \frac{H^2x}{L} \right)^{1/2} + Q \right] \, dx \]

Set \( Q = 0 \)

\[ S_a = \frac{2H}{5EI} \int_{0}^{L} x^{3/2} \, dx + \frac{2H}{3GAL^{2}} \int_{0}^{L} x^{5/2} \, dx \]

\[ \delta = \frac{2H}{5EIv^2} \left[ \frac{2}{9} \right]_{0}^{L} + \frac{2H}{3GAL^{2}} \left[ \frac{2}{5} \right]_{0}^{L} \]

\[ S_a = \frac{4HL^4}{45EI} + \frac{4}{15} \frac{HL^3}{G} \]

Let \( I = \frac{bh^3}{12} \) \[ G = \frac{2}{3} E \]

\[ A = bh \]

\[ S_a = \frac{48000}{45} \frac{H}{E_b} + \frac{100}{15} \frac{H}{E_b} = 1066.7 \frac{H}{E_b} + 6.7 \frac{H}{E_b} \]

\[ \% \delta \text{ due to shear} = \frac{6.7}{1066.7 + 6.7} \times 100 = 0.625 \% \]
APPENDIX D

MAXIMUM CORE AND SKIN SHEAR STRESSES
SKIN AND CORE SHEAR STRESSES

A core material surrounded by a thin layer of flange material, such that $E_s$, $E_c$, is shown in Figure D1.

**FIGURE D1**

**SECTION A-A**

$G_s = \frac{G_c}{E_c}$

$G_s = \frac{G_c}{E_c}$

$E_c = \frac{E_s}{E_c}$

Plain Sections Remain Plain

Shear Forces Acting Over Equivalent Areas

$F_{ce} = \gamma w_{ce} L$

$F_s = \gamma 2t l$

**SECTION A-A**

**EQUIVALENT AREAS**

$G_c A_c = G_s A_{ce}$

$w_{ce} = w_c E_c / E_s$

$F_{ce} / F_s = \frac{w_c E_c}{2 t E_s}$

(D1)

A ratio between the force in the equivalent core and flange can give an average shear stress ratio when these same forces act over the true core and flange areas, Figure D2.

**FIGURE D2**
Formula D1 can be used on an airfoil crosssection, Figure D3, at midspan to determine a ratio of skin-core shear stresses.

\[ F_{ce} : F_{t} = \frac{W_{c} E_{s}}{t_{L} E_{s}} \]

FIGURE D3

For Graphing Assume

\[ F_{ce} = 1 \text{ unit} \]

\[ F_{t_{L}} = 0.577 \quad F_{t_{R}} = 0.909 \]

\[ \gamma_{avg_{L}} = 8.24 \quad \gamma_{avg_{R}} = 8.26 \]

\[ \gamma_{avg_{s}} : \gamma_{avg_{c}} = 5.3 \quad (D2) \]

Other parallel cuts will all show that the core material resists a substantial portion of the shear load. Therefore the location of the shear center will be influenced by the shear flow distribution in both the core and skin materials.

The maximum allowable composite shear stress can be approximated for an airfoil shown in Figure D3. The airfoil shape is approximated by a rectangular shape so that the math can be used in solving for \(\gamma_{max}\). Figure D4.
Assume:
\[ \gamma_{\text{avg}} = \gamma_{s_{\text{max}}} \]

And solve (D2) and (D3) simultaneously for a maximum shear load of 10 lb. (7G's). Dimension for a rectangular crosssections equivalent to the airfoil in Figure D4 were used.

Dimensions:  
\[ t = 0.025, b = 8, w_c = 7.95, t_c = 0.95, d = 1, E_c = 10^3, E_s = 6.6 \times 10^5 \]

\[ \gamma_{\text{max}} = 180 \text{ lb/in}^2 \]
\[ \gamma_{s_{\text{max}}} = 177 \text{ lb/in}^2 \]
\[ \gamma_{c_{\text{max}}} = 3 \text{ lb/in}^2 \]

Note the skin thickness to resist \( \gamma_{s_{\text{max}}} \) was \( t = 0.025 = T_{\text{shear}} \)
APPENDIX E

MAXIMUM NORMAL SKIN STRESS
The root airfoil shape is approximated as a rectangular cross section so that the moment of inertia in the elastic flexural formula can be written as a function of skin thickness, Figure E1.

\[ I = \frac{b}{12} \left( d^3 - t_c^3 \right) + 2 \frac{tt_c^3}{12} \]

\[ t_c = d - 2t \]

\[ I = 1/12 \left( 6t^2b - 12dt^2b + 6t^3b + 2t^2a - 12d^2t^2 + 12dt^3 - 16t^4 \right) \]

\[ b = \text{Root chord} = 8 \text{ in.} \quad L = 50 \text{ in.} \]

\[ d = \text{Conservatively low root thickness} = 1/4 \text{ in.} \]

\[ I = 1/12 \left( 12.5t - 51t^2 + 67t^3 - 16t^4 \right) \]

From Table I \( G_{s_{\text{ult.}}} = 6240 \text{ psi} \)

\[ G = \frac{Mz}{I} \]

\[ \frac{G_{s_{\text{ult.}}}}{z_0} = \frac{M_{\text{max}} z_{\text{max}}}{I} = \frac{\left[ q(25) \right]^{1/4}}{1/12 \left\{ 12.5t - 51t^2 + 67t^3 - 16t^4 \right\}} = 6240 \]

\[ \frac{1/12 + 51t^2 - 12.5t + 0.1084}{0} = 0.0091 = t_{\text{Normal}} \]

\[ t = \frac{12.5 \sqrt{156 - 22.1}}{10} \]
APPENDIX F

THE STRENGTH ANALYSIS OF
THIN-UNSYMMETRICAL-TAPERED CYLINDERS
THE STRENGTH ANALYSIS OF THIN-WALLED-UNSYMMETRICAL-TAPERED CYLINDERS

This model assumed that the core was missing and only the skin remained. This restriction was necessary if the original shape of the wing was to be included in a workable mathematical model. Even with this restriction the mathematical model will still give useful information.

The following is outlined step by step procedure used in evaluating this mathematical model for deflections and stresses.

I) Assume a lift, drag, and torque distribution as a function of span position.

A) Assume a lift distribution \( L = f(y) \)
B) Assume a drag distribution \( D = f(y) \)
C) Assume a torque distribution \( T = f(y) \)

Comments: The lift, drag, and torque distributions represent the loads acting on the wing at the aerodynamic center.

II) Calculate direction cosines for aerodynamic center line.

III) Establish a set of reference axis for defining an airfoil shape at any span position.

A) Define the location and shape of the root and tip airfoils using the reference axis.
B) Calculate any other airfoil shape by using analytical geometry.

Comments: The airfoils shape was approximated by drawing line segments between 32 points defining the location of the thin skin. The wing shape is defined by drawing straight lines between corresponding points defining the root and tip airfoil shapes.

IV) At any spanwise airfoil section

A) Determine the location of the aerodynamic center
   1) Define the location of the root and tip aerodynamic center.
   2) Calculate any other aerodynamic center by using analytic geometry.

Comments: Any other aerodynamic center location was defined along a
Any other aerodynamic center location was defined along a line drawn between the root and tip aerodynamic center.

B) Determine the centroid location and moments of inertia about a set of centroidal axis referenced parallel with the reference axis.

Comments: The accuracy of the centroid location and moments of inertia depend on the skin thickness. The thinner the skin the more accurate the results.

C) Determine the location of the principal centroidal axis and the moments of inertia.

Comments: The angle of rotation needed to locate the principal axis is calculated by using the moments of inertia calculated from the previous centroidal axis. This assumed principal axis is then used in calculating the product of inertia. If this product of inertia is not within a set minimum value then the axis is rotated again until this minimum value is obtained.

D) Determine the bending moment, shear force, and torque.
   1) Assume the lift, drag, and torque act at the aerodynamic center.
   2) Use the trapezoidal numerical method for calculating the torque and shear forces.
   3) Use a numerical method analogous to the trapezoidal for calculating the bending moments.
   4) Using the direction cosines previously calculated transfer the loads from the aerodynamic center line to a set of axis parallel to the free stream velocity.
   5) Using the angle of attack and chord angle the loads along the X, Z, Y axis and the XP, ZP, Y axis are calculated.

Comments: The program was generalized to handle any shaped load distribution. But interval spacing for elliptic shapes gives the most accurate results.
E) Determine the shear center location and the shear flow due to the shear forces acting along the principal centroidal axis and at the shear center.

Comments: The method of closed thin walled sections was used. The accuracy depends on how many points are used to define the shape of the thinwalled crosssection and the skin thickness. An error in shear flow values exists because the the formulas assumed nontapered beams and the shear flows were averaged and not curve fitted between points.

F) Determine the constant shear flow due to the torque acting at the shear center.

Comments: Again the accuracy depends on a thin wall approximation.

G) Determine the normal skin stresses.

Comments: Because the principal centroidal axis and the neutral axis are coincident the normal stress maybe simply calculated from the flexural formula.

H) Determine the twist and deflection

\[
\frac{d^2 \text{Displacement}}{d \text{Span}^2} = \frac{M}{EI} \quad \theta = \frac{1}{2(\text{area})G} \sum \frac{q_s}{t}
\]

Comments: The differential equation could be solved by using the fourth order runge-kutta numerical method. The flexural rigidity (EI) for the skin is equal to the flexural rigidity of the composite. Shear deformations are shown to be small with respect to deformations due to bending, Appendix C.

Unfortunately the runge-kutta and adams-multon methods of solving the differential equations were time consuming. Instead the solutions for deflections for point and distributed loads where used. Starting at the wing root the wing is divided into as many sections as desired for accuracy. Each section is treated as a freebody of constant crosssection. The deflections for each section due to moment and shear loads where accumulated using superposition as the program progressed from the root to the tip. Twisting deflections were calculated for each section as a function of the shear flow distribution in the skin due to the torque load.
**Direction Cosines for Aerodynamic Center Line**

![Diagram showing direction cosines]

**Direction Cosines Defined**

\[
\lambda = \frac{x_2 - x_1}{d}, \quad \mu = \frac{y_2 - y_1}{d}, \quad \nu = \frac{z_2 - z_1}{d}
\]

**For two lines**

\[
\cos \Theta = \lambda_1 \lambda_2 + \mu_1 \mu_2 + \nu_1 \nu_2
\]

---

**Determination of Direction Cosines**

For Aerodynamic Center Line

\[x, y, z \Rightarrow \text{Reference axis}\]

\[\lambda_1, \mu_1, \nu_1 \text{ calculated from Root and Tip A.C. locations}\]

\[
\begin{align*}
(\mu_2, \mu_3 = -\lambda_2, \lambda_3) & \Rightarrow \frac{\mu_2}{\lambda_2} = -\frac{\lambda_1}{\mu_1} \\
(\lambda_2^2 + \mu_2^2 = 1) & \Rightarrow \lambda_2 = \sqrt{\frac{1}{(\mu_2^2/\lambda_2^2) + 1}} \\
\mu_2 &= -\frac{\lambda_1}{\mu_1}, \lambda_2, \nu_2 = 0
\end{align*}
\]

\[
\begin{align*}
(\mu_2, \mu_3, \nu_2) = 0 & \Rightarrow \frac{\lambda_2}{\lambda_3} = -\frac{\mu_2}{\lambda_2} \\
(\lambda_1, \nu_3 + \lambda_1, \lambda_3 + \nu_3 = 0) & \Rightarrow \frac{\lambda_3}{\nu_3} = \frac{\nu_1 \lambda_3}{\lambda_1 \nu_3 + \mu_1} \\
(\lambda_3^2 + \mu_3^2 + \nu_3^2 = 1) & \Rightarrow \nu_3 = \sqrt{\frac{1}{(\lambda_3^2/\nu_3^2) + (\mu_3^2/\nu_3^2) + 1}} \\
(\lambda_3 = \frac{\mu_3}{\mu_1}) & \Rightarrow \lambda_3 = -\nu_3 \mu_3 / \lambda_3, \lambda_3 = \mu_3 / \lambda_1
\end{align*}
\]
AIRFOIL SHAPE DETERMINED AT ANY SPAN LOCATION

---

Equation for a line in three dimensions:

\[
\begin{align*}
\frac{x-x_a}{x_b-x_a} &= \frac{y-y_a}{y_b-y_a} = \frac{z-z_a}{z_b-z_a}
\end{align*}
\]

For line 20:

\[
\frac{x(20)-x(32)}{x(32)-x(20)} = \frac{y(20)-y(20)}{y(20)-y(20)} = \frac{z(20)-z(20)}{z(20)-z(20)}
\]

SUBROUTINE SHAPE (RX,RZ,tx,ty,rxac,rezac,txac,tyac,fx,fy,ylocat,xac,rezac,a,b)

DIMENSION RX(32), RZ(32), TX(32), TY(32), X(32), A(32), B(32)

IF (YLOCAT.EQ.YL) GO TO 30
IF (YLOCAT.EQ.YC) GO TO 40

X(I) = ((TX(I) - RX(I)) * (YLOCAT - YF) / (YL-YF)) + RX(I)

Z(I) = ((TY(I) - RZ(I)) * (YLOCAT - YF) / (YL-YF)) + RZ(I)

30 X(1)=RX(1)
    Z(1)=RZ(1)
    GO TO 50

40 X(I)=TX(I)
    Z(I)=TY(I)
    GO TO 50

50 A(I)=X(I)
    B(I)=Z(I)

60 CONTINUE

IF (YLOCAT.EQ.YF) GO TO 70
IF (YLOCAT.EQ.YL) GO TO 80

XAC = ((TXAC - RXAC) * (YLOCAT - YF) / (YL-YF)) + RXAC
ZAC = ((TYAC - RZAC) * (YLOCAT - YF) / (YL-YF)) + RZAC

GO TO 90

70 XAC = RXAC
    ZAC = RZAC
    GO TO 90

80 XAC = TXAC
    ZAC = TYAC

90 RETURN
END
DETERMINE THE LOCATION OF THE PRINCIPAL CENTROIDAL AXES

and

MOMENTS OF INERTIA

\[
\begin{align*}
\Delta x &= x(x_2) - x(x_1) \\
\Delta y &= \Delta x / 2 \\
d &= \sqrt{\Delta x^2 + \Delta y^2} \\
\sin \phi &= \frac{\Delta x}{d} \\
\cos \phi &= \frac{\Delta y}{d} \\
I_{x_2} &= \frac{d^3}{12} \\
I_{y_2} &= \frac{d^3}{12} \\
A &= d \\
\end{align*}
\]

<table>
<thead>
<tr>
<th>Problem</th>
<th>Area</th>
<th>X</th>
<th>Y</th>
<th>Ax</th>
<th>Ay</th>
<th>Axz</th>
<th>Ixz</th>
<th>Ax^2</th>
<th>Ixz^2</th>
<th>Ix</th>
<th>Iy</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>12</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>8</td>
<td>9</td>
<td>10</td>
<td>11</td>
</tr>
</tbody>
</table>

SUBROUTINE CENTRAL(x, y, z, xbar, zbar, xzbar, xbar, ybar, zbar, xac, zac, tac, tng, pm)

DIMENSION x(30), y(30), z(30)

TANG = 0.0
ANG = 0.0
AREAT = 0
AXT = 0
ARY = 0
AXET = 0
AYT = 0
DPhi = 1.0, 32.

IF (TANG < 0.0) RETURN

IF (ANG < 0.0) ZBAR = CBAR
IF (ANG > 0.0) ZBAR = CEBAR
XBAR = AXT - AREAT * (CEBAR + 2)
ZBAR = ZET - AREAT * (CZBAR + 2)
XBAR = AREAT * ZBAR
ZBAR = ZET * AREAT * XBAR
RETURN

END
NUMERICAL DETERMINATION OF THE BENDING MOMENT, SHEAR FORCE, AND TORQUE

\[ \text{Force} = \int_{a}^{b} f_{(y)} \, dy = \sum_{i=0}^{n} \frac{H}{2} (f_i + f_{i+1}) = \frac{H}{2} \left[ f_0 + 2f_1 + 2f_2 + \ldots + 2f_{n-1} + f_n \right] \text{[lb]} \]

\[ \text{Moment} = \int_{a}^{b} f_{(y)} y \, dy = \sum_{i=0}^{n} \frac{f_i + f_{i+1}}{2} \frac{y_i + y_{i+1}}{2} \text{[lb,m]}. \]

\[ = \frac{H}{4} \left[ f_0 y_0 + f_0 y_1 + f_1 y_1 \right. \\
+ 2f_1 y_1 + f_1 y_2 + f_2 y_2 \\
+ 2f_2 y_2 + f_2 y_3 + f_3 y_3 \\
+ \ldots \\
\left. \text{... } + 2f_{n-1} y_{n-1} + f_{n-1} y_n + f_n y_n \right) + f_n y_n \]
The following is quoted from Bruhn. In this solution, we determine the centroid of the internal shear flow system for bending of the closed section about axis X without twist. This point is called the shear center. The external shear load can be resolved into a shear force acting through the shear center plus a torsional moment about the shear center.

We start the solution, as shown in the example problem, by assuming the shear flow is zero at the cut section. This section will bend without twist if the external shear load acts through the shear of the open section.

The closed section will be assumed to bend without twist, and the resulting shear flow pattern will be determined.

The equation for angular twist per inch length of the beam is,

$$\Theta = \frac{1}{2AG} \sum \frac{Qs}{t}$$

where $s$ equals the length of a web or wall.

or

$$2A\Theta = \frac{1}{G} \sum \frac{Qs}{t}$$

The right hand side of this equation represents the total shearing strain around the cell which must be zero for no twist of cell. Since $G$ is constant, we can assume it as unity as only relative values of strain are needed in the solution. Thus the total shearing strain around cell is proportional to,

$$\varepsilon = \sum \frac{Qs}{t}$$
If the cell is not to twist the relative twist of $\theta$ must be cancelled by adding a constant shear flow around the cell to give a total shearing strain. This shearing strain for a constant shear flow is,

$$\theta = q \sum \frac{S}{t}$$

If this constant shear flow is added to the shear flow for the open section then the shear flow for the closed section results.

Now that the shear flow pattern for the closed section is known the shear center can be located by summing moments about some point and dividing by the shear force.

$$\bar{x} = \frac{\sum M_o}{\sum F_z}$$

The moment of the external load will cause a torque about the shear center. The resulting shear flow is calculated by

$$q = \frac{T}{2A}$$
EXAMPLE PROBLEM

Core density = 0.00116 lb/in
Skin density = 0.033 lb/in

\( E_b = 6.6 \times 10^5 \)

RESULTS

TIP DEFLECTION

Analytic 0.0132 in  
Numeric 0.0131 in  Error 0.7%

MIDSPAN CALCULATIONS

Moment of inertia about X and Z axis

Analytic 1895.2 in\(^4\)  
Numeric 1898.5 in\(^4\)  Error 0.2%

Product of inertia

Analytic 0  
Numeric 0  No difference

Shear in the X and Z direction

Analytic 100 lb  
Numeric 100 lb  No difference
Moment about the X and Z axis

Analytic  2500 in lb
Numeric   2500 in lb

Centroid location

<table>
<thead>
<tr>
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<th>Z</th>
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Shear center location

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<td>11.251</td>
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<tr>
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Shear flow distribution

Analytic  See page D12  No noticeable difference
Numeric   See computer output listing

Normal stress distribution at point 16

<p>| | | |</p>
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Weights

Core

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Skin

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<td>lb</td>
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<td>No difference</td>
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Shear Flow Distribution

Equations Used

\[ q = -\frac{V_t}{I_x} \sum \frac{\bar{z}}{t} A \quad \sum q \frac{5}{2} \quad q \frac{5}{2} \]

Open Section

\[ I_x = 189.85 \quad V_t = +100 \quad \frac{V_t}{I_x} = 0.0526731630 \]

\[ q_o = 0 \]

\[ q_o = q_o - \frac{V_t}{I_x} (-0.25) 22.5(0.25) \]

\[ q_o = +3.333223598 \]

\[ q_b = q_a - \frac{V_t}{I_x} (-5.625) 11.25(0.25) \]

\[ q_b = +4.166529497 \]

\[ q_c = \frac{V_t}{I_x} (5.625) 11.25(0.25) \]

\[ q_c = +3.333223598 \]

\[ q_d = q_c + \frac{V_t}{I_x} (11.25) 22.5(0.25) \]

\[ q_d = 0 \]

\[ q_e = q_d - \frac{V_t}{I_x} (5.625) 11.25(0.25) \]

\[ q_e = -0.8333058990 \]

Calculating Unbalance Closed Section

\[ \frac{\bar{q}}{t} = 2 \left[ \frac{q}{0.25} \frac{5}{2} + \frac{q}{0.25} (9.25) 0.66 (11.25) \right] \]

\[ \sum \frac{\bar{q}}{t} = 599.9802476 \]

Constant Shear Flow to resist Unbalance

\[ q \frac{5}{2} = -599.9802476 \]

\[ q = -1.666666667 \]
COMPUTER OUTPUT LISTING
FOR THE
EXAMPLE PROBLEM
APPENDIX G

THIN WALLED CANTILEVER BOX BEAM
DEFLECTIONS
Using the same 7G uniform lift distribution and the same moment of inertia assumed in Appendix E an approximation of the maximum wing tip deflections can be made by the formula;

\[
y_{\text{max}} = \frac{wL^4}{3EI}
\]

\[
y_{\text{max}} = \frac{0.175 (51)^4}{8 (6.6 \times 10^5)^{1/2} \{12.5t - 51t^2 + 67t^3 - 16t^4\}}
\]

\[
t = 0.01 \text{ m} \quad y_{\text{max}} = 2.4 \text{ m}
\]

\[
t = 0.025 \text{ m} \quad y_{\text{max}} = 10 \text{ m}
\]